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Source-receiver distances and redshifts inferred from luminosity distances under pure modes of expansion

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Abstract In case of expansion, the distance between sources and receivers at the time of light emission or reception are not the distance travelled by light because space expands ahead of and behind the light front. In addition, these distances cannot be deduced from redshifts without assuming a preconceived distance-redshift diagram under a given mode of expansion. Different scenarios of expansion have been calculated, based on the predicted physico-chemical maturation stages of the universe and which are combinations of a limited number of pure expansion modes. In the present compendium, the relations between redshift and distances are systematically determined in all these modes of expansion: proportional to cosmic time, power law or exponential, and for the different assumptions about the origin of the redshift: kinematic, relativistic and wave stretching effects. None of these combinations gives the same results in terms of redshift, of horizon, of distances and of deviation from the Hubble law, thereby providing toolkits for comparing observations and hypotheses.

Keywords: Space expansion; cosmological redshift; cosmological distances; informational horizon.

1 Introduction

The distance-redshift diagram initiated by Hubble [1] is a central piece of information in cosmology, which can be envisioned as the black box of the universe in which are recorded the past modes of expansion. However, to allow using this diagram for deducing expansion rates, the measurements of the redshifts and distances should not have been experimentally correlated. Notably, deducing distances from redshifts would obviously introduce circular distortions in the reasoning. The distance used in the present study is the distance travelled by light, written D_L . Among the different measurements of distances in astronomy, clearly summarized in [2], this distance is in theory equivalent to the luminosity distance, based on the decrease of apparent luminosity with distance of objects with known global luminosity. Be-

cause of space expansion, D_L is not the distance of the celestial body when the light was emitted since it was shorter, nor the actual distance because the space separating us from the star continued to expand during the time of travel of light. Hence, mathematical twists are necessary to connect distances to redshifts. These relationships are straightly established here without assuming a particular model. Instead of starting from Friedmann equations as usual, all cases of expansion will be examined using minimalist approaches applied to flat spaces devoid of gravitational influences. Let us begin with the basics: the origin of the redshift.

2 The main hypotheses on the origin of the redshift

2.1 Redshift without expansion

General relativity would be sufficient to cause distant objects to appear redshifted as a consequence of an apparent slowing of time. This attractive possibility will not be examined here, as space will be approximated as globally flat and not shaped by gravity. Another hypothesis, called "tired light", assumes that the longer is its flight, the more the light loses its energy by interaction with particles encountered along its path (which can originate from the quantum vacuum). As the tired light theory is not based on simple optical rules, it will not be treated here, but this theory curiously uses the same law as the simple exponential expansion [3], which can of course give it a misleading success. This case will be indirectly treated when studying exponential expansion.

2.2 The redshift caused by expansion

The prevailing view is that the cosmological redshift is related to space expansion, but divergences exist about this relation. Two interpretations of the redshift, Doppler effect and wave stretching will be compared.

2.2.1 The redshift interpreted as a kinetic Doppler effect

The Hubble Redshift is frequently, perhaps erroneously, interpreted as a Doppler effect conceived conventionally (with a single light emission point but not as the addition of infinitesimal Doppler shifts along the light path postulated in the theory of [2, 4]). An observer sees a red shift if the light source moved away from him when emitting the observed light. In case of expansion, the distant stars move away from each other, but passively. As mutually receding objects can belong to the same inertial reference frame, the question is whether a passive speed can generate a Doppler effect. With uniform motion, the redshift corresponding to the classical Doppler formula is $z = V_{rec}/c$ where V_{rec} is the recession velocity and the relativistic redshift based on the Doppler formula of Einstein [5] is

$$z_r = \sqrt{\frac{c + V_{rec}}{c - V_{rec}}} - 1 \quad (1)$$

This equation is little used in astrophysics in which recessional velocities can be superluminal ($V_{rec} > c$) [6]. In addition, this formula has been built in special relativity, whereas recession velocities V_{rec} are generally supposed not uniform.

2.2.2 The redshift interpreted as a stretching of waves during their travel

Before the publication of Hubble [1], Lemaître had shown that wavelengths should follow expansion [7]. For an interval of universe

$$ds^2 = dt^2 - a(t)^2 d\sigma^2 \quad (2)$$

where $d\sigma$ is the element length of a space of radius equal to 1, the equation of a light beam is

$$\sigma_2 - \sigma_1 = \int_{t_1}^{t_2} \frac{dt}{a} \quad (3)$$

where σ_1 and σ_2 are the coordinates of a source and an observer. A beam emitted later at $t_1 + \delta t_1$ and arriving at $t_2 + \delta t_2$ undergoes a shift such that

$$\frac{\delta t_2}{a_2} - \frac{\delta t_1}{a_1} = 0 \quad (4)$$

giving

$$z = \frac{\delta t_2}{\delta t_1} - 1 = \frac{a_2}{a_1} - 1 \quad (5)$$

where δt_1 and δt_2 can be considered as the periods at emission and reception respectively [7]. In his article, Lemaître called this effect a Doppler effect. This term is acceptable if broadly defining the Doppler effect as a wave distortion, but this is not the classical Doppler effect related to the speed of the source. It will be shown

here that both mechanisms give different results regardless of the type of expansion. In the wave stretching effect of Lemaître, the ratio between the reception and emission wavelengths simply follows the increase of the distance between the source and the receiver which took place during the light flight between the time point of emission (source and receiver spaced by D_E) and that of reception (source and receiver spaced by D_R):

$$\frac{\lambda^{\text{app}}}{\lambda} = \frac{D_R}{D_E} \quad (6a)$$

The corresponding redshift z_s is the relative distance increase

$$z_s = \frac{D_R - D_E}{D_E} \quad \text{or} \quad D_R = D_E(1 + z_s) \quad (6b)$$

3 Relationships between Doppler effects and distance changes in uniform motion

The simple case of uniform motion is sufficient to perceive the symmetric nature of relative motion and the absence of a static medium and the mode of calculation of distance changes. Imagine that a source and a receiver can move relative to one another and in addition, can move relative to an hypothetical static medium supporting light propagation at speed c . The source and the receiver recede from each other at speed v . At time t_E , when spaced from the receiver by D_E , the source emits a light beam propagating towards the receiver.

3.1 In an immobile medium

Different results are obtained depending on whether this is the source or the receiver which moves relatively to the background medium (Fig.1).

3.1.1 The receiver is considered immobile relative to the background medium

Light reaches the receiver at time t_R after crossing a distance D_E (bottom scheme of Fig.1). Hence, the duration of the light travel is

$$t_R - t_E = D_E/c \quad (7a)$$

At t_R , the new spacing between the source and the receiver has become

$$D_R = D_E + v(t_R - t_E) \quad (7b)$$

Replacing the duration in Eq.(7b) by its value given by Eq.(7a), yields a distance ratio corresponding to a classical Doppler effect

$$\frac{D_R}{D_E} = 1 + \frac{v}{c} \quad (7c)$$

Distance increases in the same ratio that the classical Doppler effect. In case of collinear approach, the same reasoning gives

$$\frac{D_R}{D_E} = 1 - \frac{v}{c} \quad (7d)$$

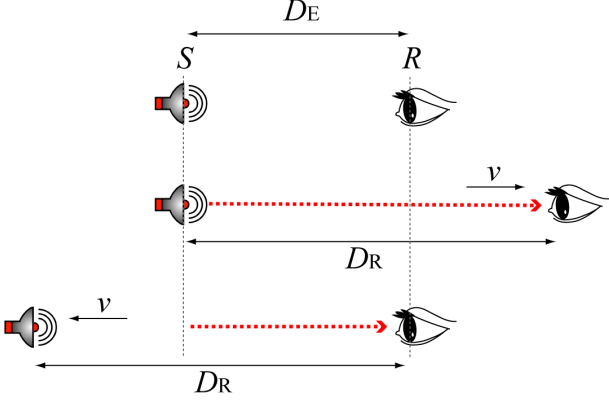


Figure 1. A light pulse is emitted by a source (S) when spaced from a receiver (R) by D_E . Just like a ball thrown between two football players, light travels through a static medium relatively to which either S (middle line) or R (bottom line), is considered immobile.

3.1.2 The source is considered immobile relative to the background medium

In this case, light is expected to reach the receiver after crossing a distance D_R (middle scheme of Fig.1). The duration of the light travel is

$$t_R - t_E = D_R/c \quad (8a)$$

Replacing the duration in Eq.(7b) by its value given by Eq.(8a), yields a conjectural Doppler formula

$$\frac{D_R}{D_E} = \frac{1}{1 - \frac{v}{c}} \quad (8b)$$

In case of collinear approach, the same reasoning gives

$$\frac{D_R}{D_E} = \frac{1}{1 + \frac{v}{c}} \quad (8c)$$

These contradictory results suggest that the notion of immobility in a static space should be ruled out. Interestingly, the geometric means of the two extreme results obtained by postulating the static medium (Eqs (7c)/(8b) and Eqs (7d)/(8c)), is the relativistic Doppler effect. For the recession:

$$\left\langle \frac{D_R}{D_E} \right\rangle = \sqrt{\frac{c+v}{c-v}} \quad (9a)$$

and for the approach

$$\left\langle \frac{D_R}{D_E} \right\rangle = \sqrt{\frac{c-v}{c+v}} \quad (9b)$$

3.2 Without background medium

In the special relativity theory, uniform motion cannot be attributed specifically to one of the relatively moving frames. The total distance crossed by light is not D_R nor D_E , but

$$D_L = c(t_R - t_E) = \frac{c}{v}(D_R - D_E) \quad (10)$$

The precise relationships between D_R , D_E and D_L will be calculated later. These simple considerations suggest that a relative speed of light should be defined, not bound to the source or the receiver.

3.3 Definition of a relative speed of light

For light following geodesics ($s = 0$), the metric of Friedmann-Robertson is very simple. By rewriting the interval of Eq.(2) in Cartesian coordinates, this metric reads

$$ds^2 = (cdt)^2 - a(t)^2(dx^2 + dy^2 + dz^2) \quad (11)$$

where $a(t)$ is the spatial expansion factor depending on time only [8]. For a light beam oriented along the x axis, this line element reduces to

$$\frac{dx}{dt} = \frac{c}{a(t)}. \quad (12)$$

The ratio would be equivalent to a relative speed if adding a unit of distance to $a(t)$. Accordingly, a time⁻¹ equation is obtained with the Friedmann-Lemaître-Robertson-Walker approach giving $d\chi/dt = c/a(t)$ where χ is an angle (without dimension) and $a(t)$ is a distance, more appropriately written $R(t)$ in this case.

4 Influence of expansion on signal connection

The relative speed of light and expansion could limit or even forbid the connection between sources and receivers. Let us examine precisely the different cases of expansion.

4.1 Uniform expansion

Several authors have suggested a mode of universe expansion called $a \propto t$, which simply follows the cosmic time [9, 10, 11]. When $a(t) = ct$, $\dot{a} = c$ and $H = 1/t$, expansion follows the age of the universe according to

$$\frac{D(t)}{D_E} = \frac{t}{t_E} \quad (13)$$

In fact, with a linear cosmic time, this expansion simply corresponds to the uniform expansion at constant speed v . Indeed, Eq.(13) can be re-written

$$\frac{D(t)}{D_E} = 1 + \frac{t - t_E}{t_E} \quad (14a)$$

or

$$D(t) = D_E + \frac{D_E}{t_E}(t - t_E) \quad (14b)$$

that is

$$D(t) = D_E + v(t - t_E) \quad (14c)$$

Uniform expansion means at constant intervals, but not necessarily slow expansion, as the speed can be unlimited. With uniform expansion and given sufficient time, a walker will inevitably cross a road even if stretching at gigantic superluminal speed, whatever his walking speed (eg walk at $c = 4$ km/h while the road stretches at $v = 1,000$ km/h). This result is so counterintuitive that it has been popularized as a mathematical game [12]. The relative speed (time^{-1}) can be written in two manners using the correspondence $D(t) = D_E(t/t_E) = D_E + vt$,

$$\frac{dx(t)}{dt} = \frac{ct_E}{D_E t} = \frac{c}{D_E + vt} \quad (15)$$

whose integration gives a unitless relative position x , starting from $x(t_E) = 0$, of

$$x(t) = \frac{ct_E}{D_E} \ln \frac{t}{t_E} = \frac{c}{v} \ln \left(1 + \frac{vt}{D_E} \right) \quad (16)$$

These functions without ceiling indicate that the walker continues its merry way and necessarily reaches the other end of the screen (when $x(t) = 1$), even with a very high stretching rate v . If the walker is a photon, signals cannot be disconnected in the universe. The journey can however be very long as its duration follows

$$\Delta t = \frac{D_E}{v} (e^{v/c} - 1) \quad (17)$$

Whatever the relative values of v and c , there is no disconnection between the different parts of this universe. To try to interrupt this connection, one must change the mode of stretching. Let us examine first a mode of expansion widely used in astrophysics, the geometric progression called "power law".

4.2 Power law expansion

This mode of expansion, predicted by calculations based on the theoretical constituents of the universe, satisfies

$$D(t) = D_E \left(\frac{t}{t_E} \right)^u \quad (18)$$

where u can take different values depending on the maturation stage of the universe described in astrophysics courses: $1/2$ for the radiation-dominated era, $2/3$ for the matter-dominated era, $1/3$ for the stiff fluid, $u = 2$ for a minimal condition of inflation satisfying $\ddot{a}/a > 0$, etc. The relative position of the walker (photon) is obtained by integration between t_E and t

$$\begin{aligned} x(t) &= \int_{\tau=t_E}^t \frac{c}{D_E} \left(\frac{\tau}{t_E} \right)^{-u} d\tau \\ &= \frac{c}{(1-u)} \frac{t_E}{D_E} \left[\left(\frac{t}{t_E} \right)^{1-u} - 1 \right] \end{aligned} \quad (19)$$

Things get interesting:

- (i) For $u < 1$, $x(t)$ increases continuously with t and therefore can exceed 1 without problem. As for the previous modes of expansion, the universe is revealed in its entirety though at different ages.
- (ii) For $u > 1$, information cannot cross a virtual limit called horizon

$$x(t_\infty) = \frac{c t_E}{D_E(u-1)} \quad (20)$$

The success of the connection depends on the initial conditions of light emission: the time of departure (t_E) and the distance to be covered as it is at this time (D_E). Now let us try another accelerated mode of expansion in which time is not raised to a power, but is itself an exponent: the exponential expansion.

4.3 Exponential expansion

With an exponential expansion of rate H , the initial distance stretches according to $D(t) = D_E e^{H\Delta t}$ and the relative speed of light is

$$\frac{dx(t)}{dt} = \frac{c}{D_E} e^{-H(t-t_E)} \quad (21)$$

whose integration between t_E and t , gives his relative position on the source-receiver spacing unit.

$$x(t) = \frac{c}{HD_E} (1 - e^{-H\Delta t}) \quad (22)$$

This position tends asymptotically to a maximum c/HD_E , which means that for given values of H and c ,

the success of the connection depends on D_E . To reach the goal, $c/H D_E$ must exceed 1, so D_E must be lower than c/H . The critical distance c/H is a disconnection point in the expanding universe, that is unreachable because the time of travel

$$\Delta t = \frac{1}{H} \ln \left(\frac{1}{1 - \frac{H D_E}{c}} \right) \quad (23)$$

would become infinite, and at t infinite, the relative speed $dx(t)/dt$ is zero. This distance is universal in a homogeneously expanding universe and valid from any starting point.

4.3.1 Consequence of the Hubble law $V_{rec} = HD$

The proportionality between the recession velocity and the distance can be written $dx/dt = Hx$, where H is the slope of a straight line. As the slope of a straight line is a constant, the unique solution of the Hubble law is $x(t) = x(0)e^{Ht}$. This expansion is not only a model (of de Sitter), but also just the only possible mathematical solution of $V_{rec} = HD$, as long as H is constant. It is recommended to write $V_{rec} = H_0 D$ to clearly indicate that $H(t)$ may have been different in the past. Interesting questions are: how much of the universe we can see and how far we have to look for seeing a H different from H_0 .

4.3.2 Specificities of the exponential mode of expansion

- (i) It is an infinite mode of expansion without big bang, because $a(t) = a_0 e^{Ht}$ (Hubble law) has no root, so a universe steadily submitted to exponential expansion has always existed. The absence of an original singularity is particularly interesting in the case of an infinite universe, because getting an infinite space by stretching a finite space is a pure mathematical impossibility.
- (ii) This mode of spatial expansion is that of the flat version of the universe of de Sitter [13] (for $k = 0$), towards which converge at long times the positively curved version ($x(t) = x(0) \cosh Ht$ for $k = 1$), and the negatively curved version ($x(t) = x(0) \sinh Ht$ for $k = -1$). After a period of success, the model of exponential expansion of de Sitter was disqualified because it has been shown to not satisfy theoretical predictions based on assumptions of pressure and density of the universe. It is however supposed to be the present mode of expansion driven by vacuum energy since we entered the "dark energy-dominated era" in which H_0 is a constant $H_0 = \sqrt{\Lambda/3}$, where Λ is the cosmological constant.

5 Tools and observables

5.1 Nomenclature

The traditional nomenclatures should be adapted to cosmology in which the suffix 0 is not used for the initial condition as in most usual treatments, but for the final condition (present). The following symbols are therefore used to avoid misunderstanding:

- t_E is the date of light emission (initial, generally written t_0 in other scientific contexts). This date can not be measured directly.
- t_R is the date of reception of the light (final). This is the present age of the universe, written t_0 by cosmologists. Of course if running time backward, an acceleration becomes a deceleration and vice versa, which may induce some misunderstandings.
- D_E is the initial distance between the star and the telescope (which did not exist yet!) at t_E . D_E is not known *a priori*.
- D_R is the distance between the source and the telescope when light enters it.
- D_L is the distance travelled by light between its emission and its reception. Hence, $D_L/c = t_R - t_E$. D_L is smaller than D_R because the fraction of space already crossed by light continued to stretch until reception. The measurement of D_L is simple, at least in its principle. In a flat 3D space, a source emitted light in all the directions so that the light front progressively increases as the surface of a sphere $4\pi D^2$ where $D = ct$. As a consequence, the flux of light I detected by an observer at the surface of this sphere corresponds to the $1/4\pi D^2$ th of the total light I_{tot} emitted by the celestial body when it was as it appears in the telescope after redshift correction, giving

$$D = \sqrt{I_{tot}/4\pi I}$$

This distance is not based on z , even if z is used in practice to assist the measurement, for correcting energy and selecting the range of observable wavelengths to consider. Type Ia supernovae are particularly appropriate for measuring distances by this way given their standard brightness, which greatly helped refining the distance-redshift diagram in the last decades [14].

5.2 Quantities measurable in our telescopes

In the theory of Lemaître, the redshift is given by the ratio D_R/D_E (Eq.(6)), but none of these distances is directly measurable, contrary to the distance crossed by light (D_L). Note that the same light received at t_R is used to measure both the redshift and the distance.

5.3 Additional recipes

t_R , the present age of the universe, is not known but can be replaced by a constant because it is common to all our measurements. Finally, a very useful tool is the fractional distance $x(t)$ crossed by the front of light between the transmitter and the receiver, defined by the function

$$x(t) = \int_{\tau=t_E}^t \frac{c}{D(\tau)} d\tau$$

where D is the source-receptor distance. Then, the date of arrival t_R is simply obtained by solving

$$x(t_R) = 1$$

6 Redshift-Distance relationships expected for the different modes of expansion

Let us examine successively the different modes of expansion and for each one, the two hypothetical causes of redshift : Doppler effect and wave stretching.

6.1 Power-law expansion

6.1.1 Classical Doppler effect

$$D_E = D_R \left(\frac{t_E}{t_R} \right)^u \quad (24a)$$

The speed of the source at emission can be deduced from the distance at reception

$$z_d = \frac{1}{c} \frac{dD_E}{dt_E} \quad (24b)$$

that is, taking D_R and t_R as constants

$$z_d = \frac{uD_R}{c} \frac{t_E^{u-1}}{t_R^u} = \frac{uD_E}{ct_E} \quad (24c)$$

replacing t_E by $t_R - D_L/c$ and D_R or D_E by their values deduced from $x(t_R) = 1$,

$$z_d = \frac{u}{1-u} \left[\left(1 - \frac{D_L}{ct_R} \right)^{u-1} - 1 \right] \quad (24d)$$

where t_R can be considered as a constant at our time scale.

6.1.2 Wave stretching

The result is straightforward

$$\frac{D_R}{D_E} = \left(\frac{t_R}{t_E} \right)^u = \left(1 - \frac{D_L}{ct_R} \right)^{-u} \quad (25a)$$

and

$$z_s = \left(1 - \frac{D_L}{ct_R} \right)^{-u} - 1 \quad (25b)$$

6.2 The particular case $u = 1$ ($a \propto t$)

As explained previously, this case merely corresponds to uniform expansion.

6.2.1 Doppler effect

Classical Doppler effect

The recession velocity for this mode of expansion is simply D/t , where D and t vary proportionally,

$$\frac{D_E}{t_E} = \frac{D_R}{t_R} = v$$

and

$$z_d = \frac{v}{c} \quad (26)$$

which supports superluminal velocities.

Relativistic Doppler effect

Using special relativity, the redshift is

$$z_r = \sqrt{\frac{c+v}{c-v}} - 1 \quad (27)$$

6.2.2 Wave stretching

By definition for this mode of expansion,

$$\frac{\lambda^{\text{app}}}{\lambda} = \frac{D_R}{D_E} = \frac{t_R}{t_E} \quad (28)$$

and by solving $x(t_R) = 1$,

$$\frac{t_R}{t_E} = e^{D_R/ct_R} = e^{v/c} \quad (29)$$

and

$$z_s = e^{v/c} - 1 \quad (30)$$

The two postulated causes of redshift give different results, which are similar only near 0 (for $v \ll c$). Both predict distance-independent redshifts, which clearly disqualifies this mode of expansion.

6.3 Exponential expansion

6.3.1 Doppler effect

Classical Doppler effect

The speed of the source at the time of light emission is

$$\frac{dD_E}{dt} = H D_E \quad (31)$$

whose value is obtained by solving $x(t_R) = 1$,

$$z_d = \frac{H D_E}{c} = 1 - e^{-H D_L/c} \quad (32)$$

As z_d is always lower than 1, the hypothesis that the redshift exclusively results from a classical Doppler effect can be eliminated for the exponential expansion.

Relativistic Doppler effect

$$z_r = \sqrt{\frac{c + H D_E}{c - H D_E}} - 1 \quad (33)$$

6.3.2 Wave stretching

The wavelengths are stretched according to

$$\frac{\lambda^{\text{app}}}{\lambda} = \frac{D_R}{D_E} = e^{H \Delta t} = e^{H D_L/c} \quad (34)$$

so that

$$z_s = e^{H D_L/c} - 1 \quad (35a)$$

whose reciprocal is

$$D_L = \frac{c}{H} \ln(1 + z_s) \quad (35b)$$

6.4 Mixing Doppler and stretching effects for the exponential expansion

The above results can be combined in a variety of manners. Let us examine only the case of the exponential expansion. The Doppler and wave stretching redshift functions established above display opposite behaviors near 0, clearly visualized by their series expansion. For the classical Doppler effect,

$$z_d \approx \frac{H_0 D_L}{c} - \frac{1}{2} \left(\frac{H_0 D_L}{c} \right)^2 \quad (36a)$$

and for the wave stretching effect,

$$z_s \approx \frac{H_0 D_L}{c} + \frac{1}{2} \left(\frac{H_0 D_L}{c} \right)^2 \quad (36b)$$

Hence, by comparison with the traditional redshift formula $z = H_0 D/c$, the Doppler effect seems to include an acceleration parameter, while the wave stretching effect seems to contain a deceleration parameter. For

nearby galaxies, the so-called peculiar or ordinary velocities can not be neglected because they can either increase, decrease or cancel the cosmological redshift. Generally, the task of astrophysicists is difficult to disentangle the different interfering sources of redshift, including gravity and peculiar velocities, and to determine the specific contribution of expansion [15]. These technical problems however demonstrate that several causes of redshift can actually interfere. After all, a recession velocity, though passive, remains a velocity and as such, one wonders why it could not give a kinetic Doppler effect contributing to the redshift. Hence, for completeness and by curiosity, let us cumulate the two effects. In addition to take the two hypotheses on the origin of the redshift into account, two ways to bring them together will be considered.

6.4.1 Combined effects

Assuming, on the one hand, that the dilation of wavelengths caused by kinetic Doppler effect (λ^{dopp}) is fixed at the emission point and then remains unchanged during light flight and assuming, on the other hand, that the emission wavelength is expanded during the trip, until received in the form λ^{app} , the cumulative effect would logically read

$$\frac{\lambda^{\text{app}}}{\lambda} = \frac{\lambda^{\text{app}}}{\lambda^{\text{dopp}}} \frac{\lambda^{\text{dopp}}}{\lambda} \quad (37)$$

using the classical Doppler effect,

$$z_{d+s} = (1 + z_d)(1 + z_s) - 1 = 2z_s \quad (38)$$

6.4.2 Additive effects

As wavelengths are lengths, they should perhaps be handled as such. Adding the relative length increase caused by Doppler effect z_d and that caused by stretching z_s gives

$$z_{d+s} = z_d + z_s \quad (39)$$

Using the classical Doppler effect,

$$z_{d+s} = e^{H D_L/c} - e^{-H D_L/c} = 2 \sinh \frac{H D_L}{c} \quad (40)$$

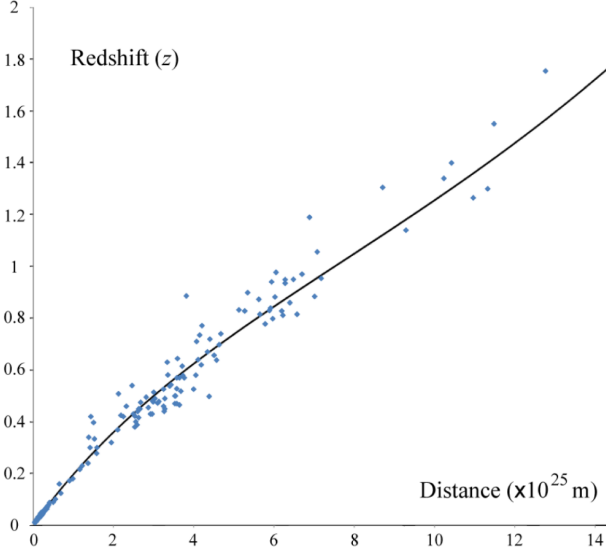


Figure 2. Speculative curve fitting of the distance-redshift supernovae distribution (dots) with an exponential expansion model in which the Doppler effect predominates at short distances while wave stretching predominates at long distances (plain line). The redshift data are from [16] and include 186 supernovae. The fitting parameters are $c = 3 \times 10^8$ m/s and $H = 3.23 \times 10^{-18}$ /s, a poorly precise value due to the conversion of Distance modulus (μ) in a non-logarithmic scale using the relationship $D_L = 10^{1+\mu/5} 10^{-6}$ Mpc.

The redshift of dual origin of Eq.(40) gives a straighter redshift-distance diagram compared to its individual components (clearly illustrated by the cancellation of the squared terms in Eq.(36)). But if for some reason the Doppler effect predominates for nearby galaxies, a bend would appear in the Hubble diagram for low redshifts, mimicking a recent acceleration for galaxies of redshift below unity. A simple function describing such hypothetic situation is $z = 2 \sinh(x)/(1+x)$ with $x = HD_L/c$. Its comparison with the observed redshifts of supernovae compiled in [16], is represented in Fig.(2), drawn in linear coordinates because a logarithmic scale is poorly appropriate to visualize the straightness of a linear plot. This exercise of redshift addition at H_0 constant is purely formal because the accepted explanation of this bend is a recent change of H driven by dark energy [14].

7 Distance of a source inferred from its apparent distance

The observed distance of cosmic objects (D_L) does not correspond to any real distance, but the actual distances can be deduced from this measured distance provided a mode of expansion is selected.

— **For the uniform expansion:**

Since $D_L = c\Delta t$, Eq.(17) gives

$$D_E = D_L \frac{v/c}{e^{v/c} - 1} \quad (41a)$$

and since $D_R = D_E + v\Delta t$,

$$D_R = D_L \frac{v/c}{1 - e^{-v/c}} \quad (41b)$$

These distances satisfy both the established distances relationship of Eq.(10) and the stretched shift of Eq.(30).

— **For the geometric expansion:**

When $x(t_R) = 1$, Eq.(19) gives

$$D_E = \frac{ct_R - D_L}{1 - u} \left[\left(1 - \frac{D_L}{ct_R} \right)^{u-1} - 1 \right] \quad (42a)$$

so that

$$D_R = \frac{ct_R}{1 - u} \left[1 - \left(1 - \frac{D_L}{ct_R} \right)^{1-u} \right] \quad (42b)$$

— **For the exponential expansion:**

When $x(t_R) = 1$, Eq.(22) gives

$$HD_E = c(1 - e^{-H\Delta t}) \quad (43a)$$

with $D_L = c\Delta t$,

$$D_E = \frac{c}{H} \left(1 - e^{-HD_L/c} \right) = \frac{cz_d}{H} \quad (43b)$$

and

$$D_R = \frac{c}{H} \left(e^{HD_L/c} - 1 \right) = \frac{cz_s}{H} \quad (43c)$$

8 Conclusion

This study expands that of Harrison [15] which rationally derived the links between redshift and distance in a purely mathematical manner depending on the type of expansion. A series of conclusions can already be drawn: For exponential expansion, the traditional redshift formula $z = H_0 D/c$ is close to $z_s = e^{H_0 D_L/c} - 1$ only in the vicinity of $D = 0$ and $z = H_0 D/c$ is valid only for the invisible actual distance D_R but not for the luminosity distance D_L . Whatever the expansion modes, the two postulated origins of the redshift: conventional Doppler and wave stretching effects, always give different results, as stated by Harrison [15]. The mere existence of redshifts higher than 1 is sufficient to rule out the Doppler

effect as the sole cause of redshift in the case of the exponential expansion. Finally, the present calculations show that the only two modes of expansion which are capable of generating disconnection barriers, are the exponential expansion (Horizon at $D_E = c/H$) and the geometric expansion if $u > 1$ (Horizon at $D_E = ct_E/(u - 1)$).

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Table 1 – Summarized consequences of the modes of expansion on the redshift, the horizon and the source-receiver distances. V_{rec} is the apparent source velocity at emission. z_d and z_s are the redshifts calculated as Doppler or wave-stretching effects respectively. D_E is the source-receiver distance at emission, D_R is the source-receiver distance at reception and D_L is the apparent distance actually measured. t_R is the present time, which can be approximated as a constant as it is identical for all our measurements.

Mode	$a \propto t$ ($u = 1$)	Power law ($u \neq 1$)	Exponential
$D(t) =$	$D_E \left(\frac{t}{t_E} \right)$	$D_E \left(\frac{t}{t_E} \right)^u$	$D_E e^{H(t-t_E)}$
$\frac{\dot{a}}{a}$	$\frac{1}{t}$	$\frac{u}{t}$	H
Horizon	No	$u < 1$: No $u > 1$: for $D_E \geq \frac{c t_E}{u-1}$	for $D_E \geq \frac{c}{H}$
V_{rec}	$\frac{D}{t} = v$	$\frac{u D_E}{t_E}$	$H D_E$
z_d	$\frac{v}{c}$	$\frac{u}{1-u} \left[\left(1 - \frac{D_L}{c t_R} \right)^{u-1} - 1 \right]$	$1 - e^{-H D_L / c}$
z_s	$e^{v/c} - 1$	$\left(1 - \frac{D_L}{c t_R} \right)^{-u} - 1$	$e^{H D_L / c} - 1$
D_E	$D_L \frac{v/c}{e^{v/c} - 1}$	$\frac{c t_R}{1-u} \left(1 - \frac{D_L}{c t_R} \right) \left[\left(1 - \frac{D_L}{c t_R} \right)^{u-1} - 1 \right]$	$\frac{c}{H} (1 - e^{-H D_L / c})$
D_R	$D_L \frac{v/c}{1 - e^{-v/c}}$	$\frac{c t_R}{1-u} \left[1 - \left(1 - \frac{D_L}{c t_R} \right)^{1-u} \right]$	$\frac{c}{H} (e^{H D_L / c} - 1)$